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Radiational Flux due to primary Scattering falling on the Horizontal Plane in the High Atmosphere.

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Abstract

In the first paper (Ref. 1) named by the same title (I) as the present the author has computed the intensity of scattering at 5 Km level in four wavelength ranges, each of has the partial energy equal to 1/4 of the total solar energy falling on the upper limit of the earth's atmosphere. In the second paper (Ref. 2) he has investigated the same problem at 75 levels from 1 Km to 38Km height, the interval between adjacent two levels being 500m. Moreover the total solar energy is divided into 12 wavelength ranges, each of which has the partial energy equal to 1/12 of the total. The study is restricted only to the primary in the plane normal to the vertical plane passing through the sun's centre in the sky dome. In the third paper (Ref. 3) he has investigated the primary scattering intensity falling on the horizontal plane of 1 cm at the above mentioned 75 level points in the case of the sun's altitude $h=30°$, $60°$, $90°$, and found the next four laws, here q being connected with the height of level points by $H=1/2(q+1)$, and λ_i' the wavelength.

1. The intensity decreases with increasing q for each h and each λ_i'.
2. The intensity increases with increasing h for each q and each λ_i'.
3. The value of the wavelength in which the intensity becomes max. decreases with increasing q for each h.
4. The values in the same meaning as 3. decreases with increasing h for each q.

In the paper just mentioned, the energy of the Sun outside the earth's atmosphere for λ_i' has been gained from the reserch of Abbot. Let θ_1 be the angle between a line passing through E_g, which is hereafter named by θ_1 line, and 0 E_q line. Let A be the azimuth of the vertical plane containing θ_1 line relative to the Sun's side. Taking the following values of θ_1 and A : $\theta_1=90+5n$ $(n=1\sim6)$, 150, 180 ; $A=0$, $\pi/2$, and $3\pi/2$, we have computed the primary scattering intensities coming from (θ_1, A_1) directions by the combination of all values of θ_1 and A mentioned above. On the foundation on this

computation we have further computed the horizontal intensity by the primary scattering in each of 12 wavelength domain falling on the upper side of E plane for the sun's altitude $h=30°$, $60°$ and $90°$.

In the fourth paper, i. e. this paper, he has investigated the same problem taking the next values of θ_1 and A :

$\theta_1=120$, 150 ; $A=0$, $\pi/2$, and $3\pi/2$, and compared the result of the 3rd with that of the fourth. We have found that the former result is greater than the latter for every combination of q, λ_i' and h.

1. Introduction. In the preceding paper the author has researched the horizontal primary scattering intensity falling on the earth's surface (Ref. 1).
In this paper he has computed the values falling on the upper sides of the horizontal planes at the level of $(q+1)/2$Km, $q=1\sim 75$.

2. The method of computation. Let O and O′ be the earth's centre and a point on its surface. Take 75 points $E_1 E_2 \cdots\cdots E_q \cdots E_{75}$ on the prolonged line of OO′, i. e. the vertical line at O′. The elevation of E_1 from O′ is 1 Km, and two adjacent points E_q and E_{q+1} are apart frcm each other by 500m. Hence, $O'E_q=\frac{1}{2}(q+1)$ Km. Consider a horizontal plane of 1 cm^2 area on E_q point, which is vertical to OE_q. It is named by E_q plane. This plane can receive the primary scattering intensity generated by the atmosphere in the sky portion bounded by a horizontal plane at E_q and the atmospheric upper limit.
The intensity is of course the horizontal primary scattering intensity falling on the upper side of E_q plane. Now, we divide the total energy of the Sun outside the earth's atmosphere into twelve domains. Let p_i, λ_i, λ_i', k and $I_o(\lambda_i')$ be respectively the mean transmission coefficient of each domain, the upper limit of wavelength of the domain, the wavelength corresponding to p_i, extinction coefficient corresponding to p_i and the Sun's energy outside the earth's atmosphere. We have the following table (Ref. 1).

The values of p_i, λ_i, λ_i', $I_o(\lambda_i')$, k_i

i	1	2	3	4	5	6
p_i	0.600	0.795	0.867	0.912	0.941	0.961
λ_i	0.409	0.466	0.519	0.577	0.638	0.708
λ_i'	0.3572	0.4364	0.4910	0.5445	0.6088	0.6763
$I_o(\lambda_i')$	1655	2806	3109	2799	2643	2321
k_i	$0.4924\cdot10^{-3}$	$0.2210\cdot10^{-3}$	$0.1380\cdot10^{-3}$	$0.8915\cdot10^{-4}$	$0.5836\cdot10^{-4}$	$0.3832\cdot10^{-4}$
i	7	8	9	10	11	12
p_i	0.973	0.985	0.991	0.995	0.998	1.000
λ_i	0.793	0.905	1.058	1.282	1.738	∞
λ_i'	0.7456	0.8567	0.9850	1.1396	1.4960	2.5356
$I_o(\lambda_i')$	1923	1405	1065	457	352	74
k_i	$0.2595\cdot10^{-4}$	$0.1482\cdot10^{-4}$	$0.8519\cdot10^{-5}$	$0.4753\cdot10^{-5}$	$0.1601\cdot10^{-5}$	$0.1940\cdot10^{-6}$

$I_o(\lambda_i')$ is gained from Abbot's research and Linke's Table. The unit and wavelength width are (cal/cm.min).10^{-6} and 0.001μ.

Let θ_1 be the angle of a line passing through E_q, which is hereafter named by θ_1 line, from $0E_q$ line. Let A be the azimuth of the vertical plane containing θ_1 line relative to the Sun's azimuth. Take the following values of θ_1 and A : θ_1=120, 150, 180 in a degree unit, A= 0 , $\pi/2$, $3\pi/2$.

In this case the atmosphere model is the same as in (Ref. 1.) and its upper limit amounts to 40Km height. Let $0''$ be the intersecting point of θ_1 line by the atmospheric upper limit, and now let T_r be a point on θ_1 line which is apart from E_q by the distance of r. The amount of primary scattering dp, received at E_q point from an air portion at T_r exposed to the direct solar ray bounded by a cone of one steradian, with its axis at (θ_1A) direction and its vertex at E_q, and a shell of 1 m width with its centre at T_r becomes in the unit of the incident direct solar ray.

$$dP_1 = \left(\frac{1}{16\pi}\right) 3\, k_i \rho(T_r)(1+\cos^2\varphi)p_i^{\Sigma(T_r)} \qquad (1)$$

$$\cos\varphi = \sin\theta_1 \cos A \cos h - \cos\theta_1 \sin h \qquad (2)$$

, here $\rho(T_r)$ being the atmospheric mass in 1 m^3 at T_r, $\Sigma(T_r)$ is the sum of two traversed masses when the direct solar ray reaches T_r from the upper atmospheric limit and the scattered ray reaches E_q from T_r in the unit of whole atmospheric mass penetrated by the vertical cylinder at $0'$ (Ref. 2).

Now put

$$\rho(T_r)p_i^{\Sigma(T_r)} = S'(i\ q\ \theta_1 A\ r\ h) \qquad (3)$$

then

$$dp_1 = \left(\frac{1}{16\pi}\right) 3\, k_i (1+\cos^2\varphi) S'(i\ q\ \theta_1 A\ r\ h) \qquad (4)$$

Let us denote

$$dH_{ps} = dp_1 \sin\left(\theta_1 - \frac{\pi}{2}\right) \qquad (5)$$

Then we have only to execute multiple integration to $dH_{ps}\cos\left(\theta_1 - \frac{\pi}{2}\right)$ with respect to r from E_q to $0''$ along θ_1 line and θ_1 from $\theta_1 = \pi/2$ to π and A from A= 0 to A= 2π to find the horizontal scattering intensity H_{ps} (iqh) at E_q, i. e.

$$H_{ps}(i\ q\ h) = \int_0^{2\pi} dA \int_{\frac{\pi}{2}}^{\pi} d\theta_1 \int_{Eq}^{0''} dr \cdot dH_{ps} \cos\left(\theta_1 - \frac{\pi}{2}\right) \qquad (6)$$

The integrant in (6) is

$$\left(\frac{1}{16\pi}\right) 3\, k_i (1+\cos^2\varphi)\, S'(i\ q\ \theta_1 A\ r\ h) \sin\left(\theta_1 - \frac{\pi}{2}\right) \cos\left(\theta_1 - \frac{\pi}{2}\right) \qquad (7)$$

Hence

$$H_{ps}(iqh)=\frac{3ki}{16\pi}\int_0^{2\pi}dA\int_{\frac{\pi}{2}}^{\pi}d\theta_1(1+\cos^2\varphi)\sin(\theta_1-\frac{\pi}{2})\cos(\theta_1-\frac{\pi}{2})$$

$$\int_{E_q}^{0''}dr\,S'(iq\theta_1Arh) \qquad (8)$$

To the calculation of (8) we will depend on the numerical integration.

Let T_1, T_2 and T_3 be three points on the line section E_q0'' which devide E_q0'' into fonr equal parts and $T_0=E_q$, $T_4=0''$, and the value of $S'(iq\theta_1 Arh)$ at $T_n(n=0\sim4)$ be $S'(n)$ and for brevity

$$S(iq\theta_1Ah)=S'(0)+4S'(1)+2S'(2)+4S'(3)+S'(4) \qquad (9)$$

Then we calculate as follows by Simpson's formula

$$\int_{E_q}^{0''}dr\cdot S'(iq\theta_1Arh)=\frac{1}{3}\cdot\frac{1}{4}\cdot E_q0''\cdot S(iq\theta_1Ah) \qquad (10)$$

Now put

$$E_q0''.S(iq\theta_1Ah)=\vartheta''(iq\theta_1Ah) \qquad (11)$$

Substitute (11) and (10) in (8), we get

$$H_{ps}(iqh)=\frac{3ki}{16\pi}\int_0^{2\pi}dA\int_{\frac{\pi}{2}}^{\pi}d\theta_1(1+\cos^2\varphi)\sin(\theta_1-\frac{\pi}{2})\cos(\theta_1-\frac{\pi}{2})\cdot$$

$$\frac{1}{12}\vartheta''(iq\theta_1Ah) \qquad (12)$$

Moreover let us put

$$(1+\cos^2\varphi)\vartheta''(iq\theta_1Ah)=\vartheta'(iq\theta_1Ak) \qquad (13)$$

In the preceding calculation, E_q0'' are expressed in the unit of the earth's radius 6370Km. Hereafter we will use C.G.S unit.

as $1\,m^3=10^6 cm^3$, $1\,Km=10^5 cm$, and

$$(\frac{1}{16\pi})3ki\cdot\frac{1}{3}\cdot\frac{1}{4}\cdot6370\cdot10^5\cdot10^{-6}=3,16816k_i \qquad (14)$$

$$S_1(iq\theta_1Ah)=3.16816k_i\vartheta'(iq\theta_1Ah) \qquad (15)$$

is the amount of primary scattering intensity received at E_q point from a cone of one steradian with its axis at (θ_1A) and vertex at E_q We have calculated S_1 for all combinations of i, q, θ_1, A, h.

Then we have

$$H_{ps}(iqh)=\int_0^{2\pi}dA\int_{\frac{\pi}{2}}^{\pi}d\theta_1\cdot S_1(iq\theta_1Ah)\sin(\theta_1-\frac{\pi}{2})\cos(\theta_1-\frac{\pi}{2}) \qquad (16)$$

Strictly speaking, $\sum(Tr)$ is dependent to A because the mass traversed by the direct solar ray is evidently dependent to A although that traversed by the scattered ray is never dependent. But when the Sun's altitude $h\geqq30°$ we can recognize $\sum(Tr)$ to be independent to A with negligible error in this research, then $1+\cos^2\varphi$ is the only one existing expression in (15) that is dependent to

A. Let us define $F(i\, q\, \theta_1, A\, h)$ by (17)

$$F(i\, q\, \theta_1, A\, h)=\sin(\theta_1-\frac{\pi}{2})\cos(\theta_1-\frac{\pi}{2})S_1(i\, q\, \theta_1, A\, h) \qquad (17)$$

To calculate the horizontal scattering intensity we must in general use the next procedure: we will at first integrate F with respect to θ_1 and then integrate thus obtained result with respect to A. This procedure demands us very much labour. But we can fortunately utilize the fact that the only existing expression $(1+\cos^2\varphi)$ in (17) is dependent to A which is clear from the above explanation. This utilization enables us to save some extent of the labour.

As $S(i\, q\, \theta_1, A\, h)$ is independent to A in this case we can put $S(i\, q\, \theta_1, h)$ instead of $S(i\, q\, \theta_1, A\, h)$. From (15) we get

$$S_1(i\, q\, \theta_1, A\, h)\sin(\theta_1-\frac{\pi}{2})\cos(\theta_1-\frac{\pi}{2})=3{,}16816k_i(1+\cos_2\varphi)\cdot$$

$$E_q0''\cdot S(i\, q\, \theta_1, h)\sin(\theta_1-\frac{\pi}{2})\cos(\theta_1-\frac{\pi}{2}) \qquad (18)$$

When if we put

$$E_q0''.S(i\, q\, \theta_1, A\, h)=S_2(i\, q\, \theta_1, h) \qquad (19)$$

(17) becomes

$$S_1(i\, q\, \theta_1, A\, h)\sin(\theta_1-\frac{\pi}{2})\cos(\theta_1-\frac{\pi}{2})=3{,}16816k_i(1+\cos^2\varphi)$$

$$\sin(\theta_1-\frac{\pi}{2})\cos(\theta_1-\frac{\pi}{2})S_2(i\, q\, \theta_1, h) \qquad (20)$$

Let us put

$$3.16816k_i(1+\cos^2\omega)=f(i\, q\, \theta_1, A\, h) \qquad (21)$$

Then

$$H_{ps}(i\, q\, h)=\int_0^{2\pi}dA\int_{\frac{\pi}{2}}^{\pi}d\theta_1\cdot F(i\, q\, \theta_1, A\, h)=\int_0^{2\pi}dA\int_{\frac{\pi}{2}}^{\pi}d\theta_1\cdot$$

$$f(i\, q\, \theta_1, A\, h)S_2(i\, q\, \theta_1, h)\sin(\theta_1-\frac{\pi}{2})\cos(\theta_1-\frac{\pi}{2}) \qquad (22)$$

It is clear that

$$\int_0^{2\pi}f(i\, q\, \theta_1, A\, h)dA=\frac{\pi}{2}\{f(i\, q\, \theta_1, o\, h)+f(i\, q\, \theta_1, \frac{\pi}{2}h)$$

$$+f(i\, q\, \theta_1, \pi\, h)+f(i\, q\, \theta_1, \frac{3}{2}\pi\, h)\} \qquad (23)$$

Denote the blacket of the right hand side by $f'(i\, q\, \theta_1, h)$ in (23). Then

$$H_{ps}(i\, q\, h)=\frac{\pi}{2}\int_{\frac{\pi}{2}}^{\pi}d\theta_1\cdot f'(i\, q\, \theta_1, h)S_2(i\, q\, \theta_1, h)\sin(\theta_1-\frac{\pi}{2})$$

$$\cos(\theta_1 - \frac{\pi}{2}) \tag{24}$$

Calculate $f''(i\ q\ \theta_1\ h)$ by

$$f'(i\ q\ \theta_1\ h)\sin(\theta_1 - \frac{\pi}{2})\cos(\theta_1 - \frac{\pi}{2}) = f''(i\ q\ \theta_1\ h) \tag{25}$$

Then

$$H_{ps}(i\ q\ h) = \frac{\pi}{2}\int_{\frac{\pi}{2}}^{\pi} d\theta_1\ f''(i\ q\ \theta_1\ h) S_2(i\ q\ \theta_1\ h) \tag{26}$$

By putting

$$f''(i\ q\ \theta_1\ h) S_2(i\ q\ \theta_1\ h) = S_3(i\ q\ \theta_1\ h) \tag{27}$$

$$H_{ps}(i\ q\ h) = \frac{\pi}{2}\int_{\frac{\pi}{2}}^{\pi} d\theta_1 \cdot S_3(i\ q\ \theta_1\ h) \tag{28}$$

we have calculated $S_3(i\ q\ \theta_1\ h)$ for the above mentioned values of i, q, θ_1 and h. we have replaced the integration of $S_3(i\ q\ \theta_1\ h)$ with respect to θ_1 by numerical integration. Hence the numerical formula of H_{ps} will be

$$H_{ps}(i\ q\ h) = \frac{\pi}{2} \cdot \frac{3}{8} \cdot \frac{\pi}{2}\{S_3(i\ q, 120°, h) + S_3(i\ q, 150°, h)\} \tag{29}$$

3. The result of computation.

The computations results for each wavelength domain D_i are tabulated in Table 1, 2 and 3. in $(1/12)\cdot I_c.\ 10^{-n}$ unit. Sum up these 12 values of each domain and divide by 12, then we can get the value for the total wave, length in $I_c.\ 10^{-n}$ unit given in the notation T in Table. In the Table $q = 0$ in the column of q means the earth's surface, $q = 1$ and q means the level of 1 Km height and $(q + 1)/2$ Km, so that the value for the level of 500m height is not calculated.

The attached mark * indicates the change of common unit. Let us denote Q as the value of q at which the value changes a unit. It should be noticed that the value of n in the array of the upper side is applied to the range from $q = 0$ to $Q - 1$. However, n in the array of the lower side is applied to that from Q to 75.

The values tor λ_i' given in Table 4, 5 and 6 are not introduced in the paper.

4. Some results. We can derive the folowing laws from the Tables as far as the primary scattering is concerned.

1 : The intensity decreases with increasing q for each h and each D_i.

2 : The intensity increases with increasing h for each q and each D_i.

3 : The value of the wavelength in which the intensity becomes max. decreases with increasing q for each h.

4 : The value in the same meaning as 3. decreases with increasing h for each q.

The ratios of the values in Table 1,2 and 3 to the corresponding values in Ref. 3 (i , e , Table I in Ref. 3) are tabulated in Table 7 . 8 . 9 .
We can find the next laws existing from this research :

1 : The value decreases with increasing wavelength for each h and q.

2 : It decreases with increasing q for each h and wavelength.

3 : It increases with increasing h for each q and wavelength.

(End)

References

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2) Sato. T., 1964 : On the scattering of the sun's ray in the high atmosphere (II). Ibid 42, 163—172

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Table 1. h = 30°

D_i / n / q	1	2	3	4	5	6	7	8	9	10	11	12	T
	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−6)	(−5)
0	8079	6078	4447	3157	2192	1497	1038	606	352	198	66	81	2315
1	7934	5622	4039	2838	1956	1329	920	535	311	174	59	71	2145
2	7796	5379	3836	2680	1844	1251	864	502	291	163	55	67	2061
3	7649	5143	3640	2532	1737	1175	811	471	273	153	51	63	1975
4	7497	4918	3452	2391	1634	1103	760	441	256	143	48	59	1892
5	7314	4687	3267	2252	1535	1036	713	413	239	134	45	55	1808
6	7115	4462	3086	2121	1442	970	668	386	224	125	42	51	1725
7	6913	4243	2915	1996	1354	910	626	361	209	117	39	48	1644
8	6683	4024	2753	1874	1271	853	585	338	196	109	37	45	1564
9	6453	3817	2597	1765	1193	799	548	316	183	102	34	42	1487
10	6227	3615	2449	1657	1118	748	513	295	171	95	32	39	1389
11	5998	3432	2305	1557	1047	700	480	276	160	89	30	36	1343
12	5772	3248	2168	1461	981	656	448	258	149	83	28	34	1274
13	5541	3064	2043	1373	919	613	419	241	139	78	26	31	1207
14	5309	2893	1917	1284	859	572	392	225	130	72	24	29	1142
15	5067	2721	1798	1203	803	534	365	210	121	67	22	27	1078
16	4843	2573	1687	1126	750	500	341	196	113	63	21	25	1010
17	4608	2419	1580	1052	701	466	317	182	105	58	19	24	961
18	4369	2265	1480	980	654	434	296	170	98	54	18	22	903
19	4139	2122	1380	916	609	403	275	158	90	50	17	20	848
20	3917	1986	1292	851	567	375	255	146	84	47	15	19	793
21	3693	1856	1203	791	524	347	237	136	78	43	14	17	748
22	3468	1733	1116	735	488	322	218	125	72	40	13	16	696
23	3255	1612	1034	682	451	299	203	116	67	37	12	15	650
24	3051	1498	962	632	418	276	188	107	62	34	11	14	605
25	2856	1387	891	584	386	255	174	99	57	32	10	13	562
26	2662	1291	805	536	357	236	161	91	53	29	9	12	520
27	2485	1196	763	500	330	218	148	84	48	27	9	11	485
28	2316	1111	709	462	305	202	137	78	45	25	8	10	451
29	2159	1027	656	428	282	187	127	72	41	23	7	9	418
30	2014	952	605	397	260	172	117	66	38	21	7	8	388
31	1875	888	563	367	242	159	108	61	35	19	6	8	361
32	1741	822	521	339	223	147	9997*	57	32	18	620*	75*	334
33	1622	761	482	314	206	136	9224	53	30	17	572	69	310
34	1506	705	446	290	191	126	8528	49	28	15	528	64	287
35	1395	651	411	268	176	116	7860	45	25	14	487	59	266
36	1296	603	380	248	163	107	7261	42	24	13	450	54	246
37	1198	557	351	228	150	9868*	6697	38	22	12	414	50	227
38	1111	515	324	211	138	9107	6182	35	20	11	382	46	210
39	1029	476	299	194	128	8398	5694	33	18	10	352	42	194
40	954	439	276	179	118	7750	5249	30	17	964*	325	39	180
41	882	406	255	165	108	7144	4845	28	15	888	299	36	166
42	816	374	235	153	100	6568	4462	26	14	818	276	33	155
43	753	345	217	140	9221*	6054	4107	24	13	752	253	30	142
44	691	317	198	129	8438	5551	3773	22	12	691	233	28	130
45	640	293	184	119	7812	5129	3482	20	11	638	215	26	120
46	590	270	169	110	7186	4718	3203	18	10	586	198	24	111
47	543	247	155	101	6591	4328	2945	17	966*	539	182	22	102
48	501	228	143	9255*	6074	3988	2701	15	886	494	167	20	9361*
49	460	209	131	8490	5557	3649	2478	14	813	454	153	18	8586
50	424	192	120	7772	5088	3361	2276	13	747	417	140	17	7893
51	388	176	110	7127	4696	3084	2088	12	685	382	129	15	7144
52	358	161	101	6529	4274	2806	1901	11	623	348	117	14	6642
53	328	148	9290*	6005	3811	2582	1748	998*	574	320	108	13	6087
54	299	135	8479	5481	3588	2357	1596	911	524	292	98	11	5562
55	274	124	7724	4993	3270	2158	1461	834	479	267	90	10	5091
56	251	113	7032	4570	2992	1964	1330	759	436	243	82	9	4652
57	228	102	6395	4156	2721	1786	1210	691	397	221	74	9	4222
58	208	9363*	5848	3778	2473	1624	1100	628	361	201	67	8	3854
59	188	8545	5345	3456	2262	1476	1000	571	328	183	61	7	3503
60	171	7697	4812	3111	2037	1338	906	517	297	166	55	6	3170

q \ D_i	1	2	3	4	5	6	7	8	9	10	11	12	T
61	154	6955	4345	2810	1841	1209	819	467	268	150	50	6	2860
62	140	6285	3927	2537	1661	1091	739	421	242	135	45	5	2591
63	125	5627	3516	2272	1487	976	661	377	217	121	40	4	2317
64	112	5046	3153	2038	1338	876	593	338	194	108	36	4	2077
65	9973*	4488	2805	1813	1187	779	528	301	173	96	32	3	1848
66	8876	3996	2495	1612	1055	693	469	268	154	85	28	3	1645
67	7846	3528	2202	1423	931	612	414	236	136	75	25	3	1453
68	6882	3095	1932	1248	817	537	363	207	119	66	22	2	1274
69	5970	2686	1677	1083	709	466	315	180	103	57	19	2	1105
70	5178	2325	1451	938	614	403	273	155	89	50	16	2	958
71	4412	1981	1236	799	523	343	233	132	76	42	14	1	816
72	3699	1659	1036	670	438	288	195	111	64	35	12	1	684
73	3043	1366	853	551	361	237	160	91	52	29	9	1	563
74	2437	1094	683	441	289	190	128	73	42	23	7	0	451
75	1873	840	525	339	222	146	99	56	32	18	6	0	346
q / n	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)

Table 2. h = 60°

q \ D_i	1	2	3	4	5	6	7	8	9	10	11	12	T
n	(−4)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)
0	1094	7177	5038	3488	2385	1612	1112	644	373	209	70	8	2822
1	1042	6549	4536	3119	2122	1429	983	569	329	185	62	7	2526
2	1012	6240	4296	2942	1997	1343	923	534	309	173	58	7	2412
3	980	5935	4065	2772	1879	1262	866	500	290	162	54	6	2300
4	946	5628	3834	2609	1763	1183	811	469	271	152	51	6	2187
5	913	5342	3621	2448	1657	1110	761	439	253	142	48	5	2080
6	879	5060	3411	2306	1554	1039	712	410	237	133	44	5	1975
7	846	4792	3215	2167	1458	973	666	384	222	124	42	5	1876
8	810	4524	3025	2034	1366	911	623	359	207	116	39	4	1776
9	776	4280	2846	1910	1281	854	583	336	194	108	36	4	1682
10	742	4043	2680	1798	1200	799	546	314	181	101	34	4	1593
11	710	3817	2517	1683	1123	747	510	293	169	94	31	3	1507
12	677	3595	2366	1575	1050	699	477	274	158	88	29	3	1424
13	646	3388	2220	1475	984	654	446	256	147	82	27	3	1346
14	614	3190	2082	1381	919	610	416	239	137	77	26	3	1320
15	581	2997	1949	1292	858	569	388	223	128	71	24	2	1192
16	553	2815	1829	1208	802	531	362	207	119	67	22	2	1125
17	523	2642	1707	1127	748	495	337	193	111	62	21	2	1057
18	495	2473	1598	1051	697	461	314	180	103	58	19	2	992
19	467	2310	1487	981	648	428	292	167	96	53	18	2	929
20	439	2158	1386	910	603	399	271	155	89	50	16	2	869
21	411	2008	1291	846	560	371	251	144	82	46	15	1	811
22	384	1868	1198	785	519	342	233	133	76	43	14	1	755
23	360	1742	1111	728	481	317	216	123	71	39	13	1	695
24	336	1615	1030	673	445	293	199	114	65	36	12	1	654
25	313	1495	950	624	411	271	184	105	60	33	11	1	606
26	291	1386	881	576	380	251	170	97	56	31	10	1	562
27	271	1285	818	532	351	231	157	89	51	28	9	1	522
28	252	1187	756	493	324	213	145	83	47	26	8	1	484
29	234	1103	700	456	299	198	134	76	44	24	8	1	449
30	218	1023	645	420	277	182	123	70	40	22	7	92*	416

D_i \ q	1	2	3	4	5	6	7	8	9	10	11	12	T
31	203	949	599	390	256	169	115	65	37	21	7	86	387
32	187	878	554	360	237	156	106	60	34	19	6	79	357
33	174	811	511	333	219	144	97	55	32	17	6	73	331
34	162	752	473	308	202	133	90	51	29	16	5	67	307
35	149	694	437	284	186	123	83	47	27	15	5	62	283
36	139	642	404	262	172	113	76	43	25	14	4	57	262
37	129	593	373	242	159	104	70	40	23	13	4	53	243
38	119	547	344	223	146	9634*	65	37	21	12	4	49	224
39	110	505	318	206	135	8886	60	34	19	11	3	45	207
40	101	467	273	190	125	8189	55	31	18	10	3	41	190
41	9414*	430	270	175	115	7550	5127*	29	17	939*	316*	38	176
42	8685	397	249	161	106	6953	4722	27	16	865	291	35	163
43	8030	365	229	149	9744*	6398	4333	25	14	796	268	32	150
44	7400	337	211	137	8983	5898	4001	23	13	732	247	29	138
45	6812	310	194	126	8255	5420	3671	21	12	673	227	27	127
46	6297	286	179	116	7592	4985	3383	19	11	619	209	25	117
47	5792	263	165	106	6982	4595	3112	18	10	570	192	23	108
48	5319	241	151	9804*	6418	4214	2854	16	936*	522	176	21	9914*
49	4872	222	139	8971	5872	3856	2618	15	859	479	162	19	9097
50	4495	204	127	8236	5410	3552	2406	14	789	440	148	17	8368
51	4117	186	117	7530	4944	3259	2207	13	724	404	136	16	7663
52	3782	171	107	6899	4516	2965	2008	12	659	367	124	15	7027
53	3467	157	9817*	6346	4154	2729	1848	11	606	338	114	13	6450
54	3163	143	8962	5793	3801	2491	1687	963*	553	309	104	12	5884
55	2893	131	8164	5278	3472	2280	1544	882	506	282	95	11	5379
56	2663	119	7470	4830	3162	2176	1406	803	461	257	86	10	4933
57	2409	108	6799	4393	2876	1888	1279	730	419	234	78	9	4467
58	2196	9866*	6180	3993	2614	1716	1162	664	381	212	71	8	4069
59	1990	8990	5616	3629	2375	1560	1056	603	346	193	65	7	3695
60	1802	8109	5091	3290	2153	1414	958	547	314	175	59	7	3345
61	1634	7362	4599	2971	1945	1277	865	494	284	158	53	6	3030
62	1475	6641	4145	2681	1755	1152	780	445	256	142	48	5	2734
63	1321	5947	3715	2401	1572	1032	698	399	229	128	43	5	2448
64	1184	5332	3328	2153	1410	925	627	358	205	114	38	4	2195
65	1054	4744	2965	1916	1254	823	558	318	183	102	34	4	1954
66	938	4217	2636	1703	1115	732	496	283	162	90	30	3	1738
67	828	3722	2327	1504	984	646	438	250	143	80	26	3	1534
68	726	3265	2041	1319	863	567	384	219	126	70	23	2	1345
69	631	2838	1772	1145	749	492	332	190	109	61	20	2	1168
70	546	2450	1529	988	647	425	288	164	94	52	17	2	1010
71	465	2086	1302	842	551	362	245	139	80	44	15	1	860
72	389	1748	1091	705	461	303	205	117	67	37	12	1	720
73	322	1446	901	582	381	250	169	96	55	31	10	1	595
74	256	1150	718	464	304	199	135	77	44	24	8	1	474
75	197	887	553	358	234	154	104	59	34	19	6	0	365
q / n	(−6)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)

Table 3. h = 90°

D_i / n / q	1	2	3	4	5	6	7	8	9	10	11	12	T
	(−4)	(−5)	(−5)	(−5)	(−5)	(−5)	(−5)	(−6)	(−6)	(−6)	(−7)	(−7)	(−5)
0	1181	7532	5241	3614	2464	1661	1144	6627	3844	2155	7282	886	2968
1	1114	6845	4705	3221	2188	1472	1011	5854	3400	1900	6422	780	2647
2	1082	6517	4458	3044	2060	1383	949	5491	3177	1781	6016	731	2528
3	1047	6200	4216	2867	1938	1299	891	5150	2979	1670	5640	686	2410
4	1008	5872	3985	2694	1818	1217	834	4821	2787	1559	5276	640	2290
5	970	5568	3746	2535	1706	1141	782	4511	2609	1460	4931	599	2174
6	933	5261	3533	2379	1602	1069	732	4221	2440	1366	4608	560	2063
7	895	4990	3326	2236	1502	1001	686	3949	2282	1278	4316	524	1157
8	858	4710	3133	2100	1407	938	641	3695	2133	1194	4031	490	1855
9	821	4450	2946	1970	1318	877	600	3457	1994	1117	3765	457	1756
10	784	4197	2770	1850	1234	821	561	3229	18 5	1044	3519	427	1661
11	747	3958	2600	1731	1156	769	525	3017	1741	973	3287	399	1568
12	712	3731	2443	1625	1081	719	491	2819	1627	909	3068	373	1482
13	676	3511	2297	1520	1013	672	458	2633	1518	848	2867	348	1397
14	642	3304	2156	1424	945	627	428	2456	1416	792	2670	324	1317
15	609	3103	2015	1330	884	586	399	2292	1319	739	2489	302	1232
16	577	2916	1887	1244	825	546	372	2131	1230	689	2322	282	1166
17	546	2734	1766	1161	770	509	347	1989	1146	641	2158	262	1096
18	515	2560	1646	1083	718	474	323	1852	1067	596	2009	244	1027
19	485	2386	1532	1008	668	441	300	1718	992	554	1865	226	961
20	456	2231	1432	937	620	410	279	1597	914	513	1735	210	899
21	427	2076	1327	873	576	379	259	1480	853	476	1703	194	838
22	399	1929	1232	807	533	351	239	1371	790	441	1487	180	780
23	373	1793	1142	748	494	325	221	1268	731	408	1375	167	726
24	348	1662	1059	692	457	300	204	1173	674	376	1268	154	674
25	325	1544	981	641	422	278	188	1083	623	348	1171	142	627
26	302	1431	906	592	390	257	175	997	576	321	1083	131	581
27	281	1322	840	547	361	238	161	922	530	297	1001	121	539
28	261	1227	777	506	333	219	148	853	490	274	922	112	499
29	243	1135	719	468	307	203	137	790	454	254	854	104	463
30	225	1052	663	432	285	187	127	729	419	234	788	96	429
31	210	975	616	400	263	173	118	673	387	216	728	88	398
32	195	903	569	370	243	160	109	621	358	200	672	82	369
33	180	834	525	342	224	148	100	573	330	184	620	75	341
34	167	772	486	316	208	137	9267*	530	304	170	573	69	316
35	155	713	449	291	191	126	8545	489	281	157	528	64	292
36	143	660	415	269	177	116	7887	451	259	145	487	59	270
37	133	609	383	248	163	107	7275	416	239	133	449	54	250
38	123	562	354	230	150	9893*	6716	384	220	123	415	50	231
39	114	520	326	211	139	9124	6180	354	203	113	382	46	213
40	105	480	301	195	128	8422	5704	326	187	105	352	42	196
41	9708*	442	278	180	118	7753	5264	301	173	665*	325	39	182
42	8963	408	256	166	109	7150	4842	277	159	888	299	36	168
43	8265	376	236	153	100	6593	4465	266	147	819	276	33	154
44	7630	347	217	141	9224*	6066	4108	235	135	754	254	30	142
45	7030	319	200	130	8497	5566	3786	216	124	693	234	28	131
46	6480	294	184	119	9816	5142	3483	199	114	638	215	26	122
47	5965	270	170	110	7170	4718	3195	183	105	585	197	23	111
48	5480	248	155	101	6590	4340	2940	168	964*	538	181	22	102
49	5050	228	143	9243*	6050	3973	2697	154	885	494	166	20	9389*
50	4633	210	131	8487	5556	3648	2480	141	810	452	152	18	8621
51	4245	192	120	7763	5096	3346	2266	129	743	415	140	17	7883
52	3884	176	110	7116	4658	3058	2071	118	679	379	128	15	7227
53	3560	161	101	6517	4266	2802	1898	108	623	347	117	14	6622
54	3249	147	9273*	5950	3895	2559	1733	989*	568	317	107	13	6050
55	2971	124	8420	5443	3565	2341	1586	905	520	290	97	11	5524
56	2722	123	7671	4960	3247	2132	1444	824	473	264	89	10	5053
57	2486	112	6982	4511	2953	1939	1313	750	431	240	81	9	4606
58	2262	102	6357	4101	2684	1763	1194	681	391	218	73	8	4191
59	2043	9236*	5770	3728	2441	1603	1085	619	356	198	66	8	3796
60	1851	8327	5231	3380	2212	1453	984	562	323	180	60	7	3436

$q \backslash D_i$	1	2	3	4	5	6	7	8	9	10	11	12	T
61	1678	7559	4722	3051	1997	1311	889	507	291	162	54	6	3111
62	1515	6819	4256	2753	1802	1183	801	457	263	146	49	5	2807
63	1356	6106	3815	2465	1614	1060	718	409	235	131	44	5	2514
64	1216	5476	3418	2211	1447	950	644	367	211	117	39	4	2255
65	1082	4872	3045	1967	1288	846	573	327	188	104	35	4	2006
66	964	4331	2707	1749	1145	752	509	290	167	93	31	3	1785
67	850	3823	2390	1544	1011	664	449	256	147	82	27	3	1575
68	746	3352	2096	1354	887	582	394	225	129	72	24	2	1381
69	648	2915	1819	1175	770	505	342	195	112	62	21	2	1200
70	560	2515	1570	1014	664	436	295	168	97	54	18	2	1037
71	477	2142	1337	864	566	371	252	143	82	46	15	1	883
72	400	1795	1120	724	474	311	211	120	69	38	12	1	739
73	330	1482	925	598	391	257	174	99	57	31	10	1	610
74	263	1181	737	476	312	205	139	79	45	25	8	1	487
75	203	910	568	367	240	158	107	61	35	19	6	0	375
q / n	(−6)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)	(−7)

Table 7. $h = 30°$

$q \backslash D_i$	1	2	3	4	5	6	7	8	9	10	11	12	T
0	0.9980	0.9372	0.9076	0.8826	0.8685	0.8594	0.8515	0.8461	0.8422	0.8401	0.8366	0.8386	0.9257
1	9898	9282	9016	8804	8663	8569	8503	8436	8405	8382	8370	8476	9260
5	9690	9106	8897	8682	8590	8499	8448	8396	8374	8357	8347	8352	9108
10	9421	8865	8722	8594	8515	8452	8424	8377	8363	8354	8341	8357	8797
15	9183	8752	8599	8526	8462	8423	8410	8372	8369	8339	8327	8351	8903
20	8980	8646	8546	8468	8438	8389	8361	8354	8350	8342	8336	8328	8686
25	8818	8530	8478	8415	8391	8361	8365	8346	8355	8333	8338	8338	8620
30	8662	8455	8403	8393	8333	8350	8357	8327	8355	8372	8335	8390	8527
35	8558	8444	8405	8375	8381	8345	8349	8349	8355	8324	8339	8353	8471
40	8488	8394	8364	8364	8369	8352	8342	8361	8357	8311	8333	8358	8451
45	8420	8379	8349	8345	8342	8332	8339	8340	8321	8334	8329	8344	8392
50	8396	8348	8333	8317	8315	8328	8319	8333	8316	8313	8312	8333	8354
55	8354	8378	8320	8304	8304	8325	8320	8347	8318	8319	8317	8295	8345
60	8341	8315	8297	8296	8294	8295	8289	8295	8294	8300	8297	8321	8316
65	8308	8284	8282	8245	8266	8261	8263	8262	8263	8272	8261	8260	8280
70	8272	8262	8250	8250	8253	8258	8248	8253	8258	8251	8245	8219	8259
75	8238	8235	8248	8248	8253	8249	8250	8258	8265	8265	8250	8202	8248

Table 8. $h = 60°$

$q \backslash D_i$	1	2	3	4	5	6	7	8	9	10	11	12	T
0	0.9936	0.9531	0.9307	0.9105	0.8971	0.8956	0.8861	0.8800	0.8770	8749	8736	0.8742	0.9681
1	9886	9461	9244	9082	8969	8881	8832	8784	8758	8743	8730	8736	9411
5	9744	9326	9132	9003	8899	8838	8798	8756	8731	8725	8708	8716	9319
10	9550	9174	9024	8920	8856	8809	8778	8745	8725	8720	8711	8721	9203
15	9356	9057	8924	8861	8800	8767	8758	8739	8725	8715	8706	8724	9085
20	9223	8951	8862	8801	8777	8769	8742	8726	8716	8711	8700	8723	9005
25	9072	8878	8796	8801	8763	8742	8720	8733	8713	8715	8697	8734	8912
30	8971	8827	8764	8732	8738	8712	8697	8730	8718	8736	8706	8720	8851
35	8765	8785	8758	8738	8712	8723	8710	8702	8705	8703	8682	8713	8762
40	8783	8778	8720	8716	8706	8708	8699	8712	8714	8707	8684	8712	8716
45	8777	8722	8700	8690	8698	8690	8689	8678	8705	8688	8697	8708	8699
50	8745	8708	8699	8697	8689	8682	8689	8671	8680	8677	8655	8700	8720
55	8703	8721	8674	8662	8689	8682	8674	8647	8674	8675	8659	8684	8700
60	8676	8654	8682	8669	8664	8664	8670	8575	8663	8666	8668	8668	8670
65	8639	8641	8639	8635	8630	8627	8638	8629	8628	8631	8628	8654	8642
70	8598	8596	8585	8584	8592	8586	8597	8584	8582	8583	8582	8560	8596

Table 9. h = 90°

q \ D_i	1	2	3	4	5	6	7	8	9	10	11	12	T
0	0.9958	0.9615	0.9414	0.9241	0.9139	0.9063	0.9015	0.8970	0.8942	8923	0.8910	0.8917	0.9943
1	9902	9512	9359	9203	9117	9047	8995	8951	8924	8916	8903	8911	9494
5	9788	9432	9259	9135	9055	9054	8958	8926	8907	8996	8883	8901	9423
10	9643	9277	9160	9073	9007	8963	8947	8915	8906	8900	8886	8902	9331
15	9471	9197	9085	9023	8975	8947	8926	8908	8894	8861	8883	8892	9232
20	9344	9099	9029	8975	8960	8932	8914	8897	8839	8875	8879	8893	9155
25	9233	9045	8992	8965	8922	8910	8868	8906	8900	8900	8878	8917	9087
30	9109	8984	8923	8907	8906	8863	8881	8890	8896	8864	8874	8901	9013
35	9064	8957	8926	8899	8884	8873	8892	8907	8892	8906	8889	8901	8985
40	9052	8922	8905	8904	8889	8883	8878	8883	8779	8898	8866	8877	8965
45	8943	8911	8889	8904	8889	8869	8898	8885	8794	8867	8897	8899	8912
50	8916	8898	8851	8881	8877	8874	8908	8868	8785	8857	8837	8894	8895
55	8877	8874	8878	8868	8866	8867	8855	8882	8773	8859	8891	8881	8871
60	8844	8840	8863	8850	8848	8849	8849	8848	8763	8855	8850	8860	8846
65	8818	8826	8816	8809	8810	8813	8815	8810	8810	8830	8805	8834	8814
70	8750	8763	8747	8749	8748	8755	8754	8750	8755	8782	8737	8723	8758

Errata

"On the Scattering of the Sun's Ray in the High Atmosphere"
of this Bulletin of No.18, 1967.

The Table must be revised as follows :

Table

q	D, $\lambda i'$	I (a)	II (a)
0	9	4184	4456
0	T	2500	4850
45	1	761	126
45	2	350	9812
45	3	220	6837
45	4	143	3990
45	5	9364	248
45	6	6160	143
45	7	4175	8029
45	8	239	3355
45	9	137	146
45	10	7664	3502
45	11	258	9085
45	12	312	231
45	T	143	277

Table

q	D, $\lambda i'$	I (b)	II (b)
0	9	4262	4539

Table

q	D, $\lambda i'$	I (c)	II (c)
1	2	7196	2019
2	2	6857	1924
1	T	2788	5409
2	T	2666	5172
60	1	2093	3463
43	8	288	4042
44	8	263	3695
45	8	243	3416
60	T	3884	753
0	10	2415	1104
1	10	2131	974
2	10	1999	914
3	10	1875	857
4	10	1752	801
5	10	1641	750
6	10	1536	702
7	10	1437	656
8	10	1344	614
9	10	1255	574
10	10	1173	536
11	10	1094	500
12	10	1022	467
13	10	954	436
14	10	888	406
15	10	834	381
16	10	775	354
17	10	721	329
18	10	671	306
19	10	623	285
20	10	578	264
21	10	535	245
22	10	496	227
23	10	459	210
24	10	423	194
25	10	391	179
26	10	373	170
27	10	334	153
28	10	309	141
29	10	285	130
30	10	264	120
31	10	243	111
32	10	225	103
33	10	207	9472*
34	10	191	8740
35	10	176	8060
36	10	163	7437
37	10	150	6860
38	10	139	6341
39	10	128	5840
40	10	118	5387
41	10	109	4974
42	10	100	4581
43	10	9230*	4218
44	10	8505	3887
45	10	7820	3574
46	10	7198	3290
47	10	6606	3019
48	10	6180	2824
49	10	5576	2548
50	10	5109	2335
51	10	4681	2139
52	10	4289	1960
53	10	3925	1794
54	10	3589	1640
55	10	3278	1498
56	10	2987	1365
57	10	2718	1242
58	10	2474	1131
59	10	2246	1027
60	10	2035	930
61	10	1839	840
62	10	1658	758
63	10	1476	675
64	10	1334	610
65	10	1189	543
66	10	1057	483
67	10	934	427
68	10	818	374
69	10	712	325
70	10	616	282
71	10	523	239
72	10	438	200
73	10	361	165
74	10	288	132
75	10	223	102

上空に於ける水平面の受くる第一次散乱光による輻射強度

上 村 静 子・佐 藤 隆 夫

地上をNo. 0（q＝0）とし，上空1KmをNo. 1（q＝1），500m上昇するごとにqを一つづつ増し，38Km上空をq＝75とする。この各高さに水平面を考え，この上面に当る第一次散乱光強度を計算した。方法は水平面とのなす角 $\theta=30°$，60°，方位角 $A=0$，$\frac{\pi}{2}$，π，$\frac{3\pi}{2}$ なる各組合せの（θ，A）方向を軸とする単位立体角内の第一次散乱光強度を基にして計算した。その結果次の4つの法則の存在することが判明した。

1. 強度は太陽の高度（h）に関係なく，各波長（λ）について高さ（q）が増すに従って減少する。
2. 強度は各高さ及び各波長につき，太陽高度が増すに従い増加する。
3. 強度が極大となる波長の値は各太陽高度につき，高さが増すに従い減少する。
4. 強度が極大となる波長の値は各高さにつき，太陽高度が増すに従い減少する。

この方法による水平面輻射量をA_1とし，$\theta=5°$，10°，15°，20°，25°，30°，60°，$A=0$，$\frac{\pi}{2}$，π，$\frac{3}{2}\pi$ の組合せにより計算した値を A_2 とするとき，A_1/A_2 の値は次の法則に従っていることが判明した。

1. 値は各h，各qにつきλの増すに従い減ずる。
2. 値は各h，各λにつきqの増すに従い減ずる。
3. 値は各q，各λにつきhの増すに従い増加する。

（以上）